Hubble constant can be interpreted as velocity equivalent of position

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Abstract: Based on a plain belief that the cosmic space should be a unified whole with universal connection, rather than a patchwork of unconnected domain blocks, we reassess Hubble law as a short-range characterization of electromagnetic interaction and Hubble constant as velocity equivalent of position. If so, it is necessary to develop a matching transformation to work for relativity, which can ensure that the causal chain is not broken and the physical description is complete. To this end, we unify space and motion into a full-velocity concept and reshape Minkowski space into a pseudo-Minkowski one confined by an unreachable physical horizon. In the reshaped, all parts are physically allowed for unobstructed communication with each other, and the transformation that keeps physical laws including Hubble law invariant, is full-velocity Lorentz transformation, under which length contraction, time dilation and historization effect are discussed. Meanwhile, as a static solution of Einstein equation, the pseudo-Minkowski universe not only can reproduce the effective results of expansion model, but also derive the background radiation temperature and Hubble constant, etc.. Especially, it provides an analytic solution of gravitational field around mass without the singularity of event horizon.

Keywords: Hubble law; Velocity equivalent of position; Strong causality; Full-velocity Lorentz transformation; Pseudo-Minkowski universe.

1 Introduction

In addition to the principle of relativity and the constancy of the speed of light, the establishment of Einstein relativity also benefited from three tacit assumptions [1]: 1) Homogeneity and isotropy—the intrinsic properties of space are the same everywhere, in any direction and for all time. 2) No memory—the extrinsic properties of rulers and clocks are functions of their current states, rather than their previous states. 3) Synchronization—all clocks at different positions of a certain frame are synchronized. Served by the three, Lorentz transformation can keep the laws of motion invariant, but excludes Hubble law. As for this electromagnetic propagation law named after Hubble, a widely accepted view is it arises from Doppler mechanism of space expansion [2,3], so that there is a linearly-receding relation $\upsilon_{\mathrm{H}} = Hr$ with Hubble constant H, defining Hubble time $\Omega = H^{-1}$ and radius $\Re = c\Omega$. Formally, Hubble law specifies the distance-dependence of frequency-shift, but in physics, it takes on the form of some immutable law related to the essential attribute of space and needs to be tested by relativistic transformation.

Einstein relativity is a product of the combination of Galileo relativity and the idea of non-instantaneous propagation of interaction, so accordingly no particle can be infinitely accelerated. And yet, a consequent question is: can a particle move infinitely far? If the answer is "no" (no factual support for the "yes" option), how will it affect relativity? The motivation of the paper is to optimize Lorentz transformation to include Hubble law. In the process, we will see: 1) What Hubble's discovery can tell us definitely is not that space is expanding, but that electromagnetic interaction acts as a short-range one. Under the clamping of gauge invariance, this short-range characteristic not only requires a set of massive electromagnetic equations of supporting P-wave radiation, but also implies a finite detectable space depth. 2) The finite depth must be specified as a constant by the principle of relativity, and even become a decisive factor of reshaping relativistic space. 3) A redefined full-velocity will occupy the position of usual velocity to characterize inertial motion, and the transformation working for relativity is full-velocity Lorentz transformation, whose physical consequences are discussed. 4) Under the dominance of Einstein field equation, a bounded pseudo-Minkowski universe without creation singularity and horizon difficulty is sketched out. Although in a static state, this universe has a traceable limited history, and can even be equivalently regarded as expanding from an extreme primitive state. Finally, it shows that pseudo-Minkowski manifold doesn't support Schwarzschild black hole, so as to avoid space being segmented by the event horizon.

2 Massive electromagnetic field equations

2.1 Extension to Maxwell equations

Historically, a set of massive electromagnetic field equations (in Heaviside units) was first proposed by Proca [4]

$$\begin{cases}\nabla\cdot\boldsymbol{E}+\dfrac{\varphi_{\mathrm{e}}}{\Re_{\mathrm{e}}^{2}}=\rho_{\mathrm{e}}, & \nabla\times\boldsymbol{E}+\dfrac{1}{c}\dfrac{\partial\boldsymbol{B}}{\partial t}=0\\ \nabla\cdot\boldsymbol{B}=0, & \nabla\times\boldsymbol{B}-\dfrac{1}{c}\dfrac{\partial\boldsymbol{E}}{\partial t}+\dfrac{\boldsymbol{A}}{\Re_{\mathrm{e}}^{2}}=\dfrac{\boldsymbol{j}}{c}\end{cases}\tag{1}$$

$\Re_{\mathrm{e}}$ denotes the force-range of electromagnetic interaction, defining a non-zero mass $m_{\gamma}=\hbar/\Re_{\mathrm{e}}c$ for photon. Due to using potentials $(\boldsymbol{A},\varphi_{\mathrm{e}})$ as observable quantities, Proca's theory loses its due gauge invariance. To recapture the lost invariance, we introduce additional vector and scalar fields $(\boldsymbol{\mathcal{B}},\mathcal{E})$, and extend Eq.(1) to 5-D Minkowski space $(ct,\xi,\zeta,\varsigma,\bar{\xi})$ with an extra-dimension $\bar{\xi}(=\Re_{\mathrm{e}}\phi)$, namely

$$\begin{cases}\text{(i) }\nabla\cdot\boldsymbol{E}+\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\mathcal{E}}{\partial\phi}=\rho_{\mathrm{e}}, & \text{(ii) }\nabla\times\boldsymbol{E}+\dfrac{1}{c}\dfrac{\partial\boldsymbol{B}}{\partial t}=0\\ \text{(iii) }\nabla\cdot\boldsymbol{B}=0, & \text{(iv) }\nabla\times\boldsymbol{B}+\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\boldsymbol{\mathcal{B}}}{\partial\phi}-\dfrac{1}{c}\dfrac{\partial\boldsymbol{E}}{\partial t}=\dfrac{\boldsymbol{j}}{c}\\ \text{(v) }-\nabla\cdot\boldsymbol{\mathcal{B}}-\dfrac{1}{c}\dfrac{\mathcal{E}}{\partial t}=\dfrac{\jmath}{c}, & \text{(vi) }\nabla\times\boldsymbol{\mathcal{B}}+\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\boldsymbol{B}}{\partial\phi}=0\\ \text{(vii) }\nabla\mathcal{E}-\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\boldsymbol{E}}{\partial\phi}+\dfrac{1}{c}\dfrac{\partial\boldsymbol{\mathcal{B}}}{\partial t}=0 & \end{cases}\tag{2}$$

Following it is a generalized conservation equation for 4-current $\vec{j}(=\boldsymbol{j},\jmath)$

$$\vec{\nabla}\cdot\vec{j}+\frac{\partial\rho}{\partial t}=0\,,\qquad \vec{\nabla}=\nabla+\frac{1}{\Re_{\mathrm{e}}}\frac{\partial}{\partial\phi}\tag{3}$$

Then, by 5-potential $A^{n}=(-\varphi_{\mathrm{e}},\boldsymbol{A},\mathcal{A})$ ($n=0,1,2,3,4$), we can express massive fields as

$$\begin{cases}\boldsymbol{E}=-\nabla\varphi_{\mathrm{e}}-\dfrac{1}{c}\dfrac{\partial\boldsymbol{A}}{\partial t}, & \boldsymbol{B}=\nabla\times\boldsymbol{A}\\ \mathcal{E}=-\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\varphi_{\mathrm{e}}}{\partial\phi}-\dfrac{1}{c}\dfrac{\partial\mathcal{A}}{\partial t}, & \boldsymbol{\mathcal{B}}=\nabla\mathcal{A}-\dfrac{1}{\Re_{\mathrm{e}}}\dfrac{\partial\boldsymbol{A}}{\partial\phi}\end{cases}\tag{4}$$

supplemented by a generalized Lorentz condition

$$\vec{\nabla}\cdot\vec{A}+\frac{1}{c}\frac{\partial\varphi_{\mathrm{e}}}{\partial t}=0\tag{5}$$

Now, it is easy to prove that the extended form has the gauge invariance under transformation of $A^{n}\to A^{n}+\chi^{,n}$, χ is an arbitrary scalar function. Based on this gauge invariance, one can always transform additional potential $\mathcal{A}$ away by properly selecting the scalar function (for example, $\chi=-\Re_{\mathrm{e}}\int\mathcal{A}\mathrm{d}\phi$). Once $\mathcal{A}$ is specified as zero, it must require additional current $\jmath=0$ to cater to charge conservation.

The performance of massive electromagnetic induction can be summarized as follows:

a) *Varying magnetic and additional scalar fields generate respectively an electric field and an additional vector field*, *described by* (ii) *and* (v).

b) *Varying electric field generates a magnetic field and an additional vector field*, *by* (iv).

c) *Varying additional vector field generates an additional scalar field and an electric field*, *by* (vii).

Due to involving more inductive forms, Eq.(2) exhibits more complicated structure. This structure encourages us to treat Hubble law as a short-range characterization of electromagnetic interaction, and then align Hubble radius $\Re$ (defined by infinite redshift $Z\to\infty$) with force-range $\Re_{\mathrm{e}}$, namely

$$r=\frac{c}{H}\frac{(1+Z)^{2}-1}{(1+Z)^{2}+1}\bigg|_{Z\to\infty}\to\Re=\Re_{\mathrm{e}}\tag{6}$$

It suggests that as natural constants deeply related to the essential properties of spacetime, the maximum possible connection distance $\Re$ and motion speed c should be incorporated into the theoretical structure of relativity.

However, looking back, the principle of relativity itself does not put forward the special requirements for connection distance and velocity threshold. Only with the cooperation of the speed limit principle can it derive a transformation to match Eq.(2), that is, the 5-D Lorentz transformation

$$\xi^n = \alpha_l^n(\gamma_u)\xi'^l , \qquad \alpha_l^n = \begin{bmatrix} \gamma_u & \beta_u\gamma_u & 0 \\ \beta_u\gamma_u & \gamma_u & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{7}$$

accompanied by a generalized velocity addition

$$\begin{cases} \upsilon_{//} = \dfrac{u+\upsilon'_{//}}{1+\beta_u\beta_{\upsilon'_{//}}}, & \upsilon_{\perp} = \dfrac{\upsilon'_{\perp}}{\gamma_u(1+\beta_u\beta_{\upsilon'_{//}})} \\ \cancel{\upsilon} = \dfrac{\cancel{\upsilon}'}{\gamma_u(1+\beta_u\beta_{\upsilon'_{//}})}, & \cancel{\upsilon}' = \sqrt{c^2-\upsilon'^2} \end{cases} \tag{8}$$

Where, γ_u is Lorentz factor with a dimensionless velocity $\beta_u = u/c$.

The path to covariant electromagnetism begins with the introduction of electromagnetic field tensor

$$F^{nl} = A^{l,n} - A^{n,l} = \begin{bmatrix} 0 & -E_\xi & -E_\zeta & -E_\varsigma & -\cancel{E} \\ E_\xi & 0 & B_\varsigma & -B_\zeta & \cancel{B}_\xi \\ E_\zeta & -B_\varsigma & 0 & B_\xi & \cancel{B}_\zeta \\ E_\varsigma & B_\zeta & -B_\xi & 0 & \cancel{B}_\varsigma \\ \cancel{E} & -\cancel{B}_\xi & -\cancel{B}_\zeta & -\cancel{B}_\varsigma & 0 \end{bmatrix} \tag{9}$$

By which, the homogeneous and non-homogeneous equations in Eq.(2) can be written as

$$F^{nl,q} + F^{lq,n} + F^{qn,l} = 0 , \qquad F^{nl}_{,l} = \frac{j^n}{c} \tag{10}$$

For the electromagnetic stress of j^n in massive fields, we have

$$f^n = \Pi^{nl}_{,l} , \qquad \Pi^{nl} = \eta_{oq}F^{no}F^{lq} - \frac{1}{4}\eta^{nl}F^{oq}F_{oq} \tag{11}$$

$\eta_{nl}(=\mathrm{Diag}(-c^2,1,1,1,1))$ is 5-D Minkowski metric, Π^{nl} the electromagnetic stress-energy tensor. When additional fields $(\cancel{B},\cancel{E})$ are ignored, all usual results are automatically restored.

2.2Massive electromagnetic waves

With the help of Eq.(4), we can extract d'Alembert equation from Eq.(2), that is

$$\vec{\nabla}^2 A^n - \frac{1}{c^2}\frac{\partial^2 A^n}{\partial t^2} = -\frac{j^n}{c} \tag{12}$$

yielding a vacuum solution

$$A^n = A_0^n \mathrm{e}^{ik_l\xi^l} , \qquad k_l = \left(\frac{\omega}{c}, \boldsymbol{k}, \frac{\cancel{k}}{\Re}\right), \qquad \cancel{k} = \frac{2\pi}{2\pi} \tag{13}$$

$\cancel{k}$ denotes the ϕ-component of wave vector. The typical aspect of massive photon is its frequent dependence

$$\omega^2 = c^2\kappa^2 , \qquad \kappa = \frac{2\pi}{\Delta} = \sqrt{k^2 + \frac{\cancel{k}^2}{\Re^2}} \tag{14}$$

κ, Δ are the generalized wave vector and length. The frequent-dependence determines that the generalized group and phase velocities always take the same value, namely

$$C_g = \frac{\mathrm{d}\omega}{\mathrm{d}\kappa} = c , \qquad C_p = \frac{\omega}{\kappa} = c \tag{15}$$

When viewed in space (ξ,ζ,ς), the group velocity is displayed as the direct projection of photon motion, and the phase velocity interpreted as the moving velocity of the intersection point of wave surface and projection axis (i.e.

ξ - axis, see **Fig**.1). So, we have

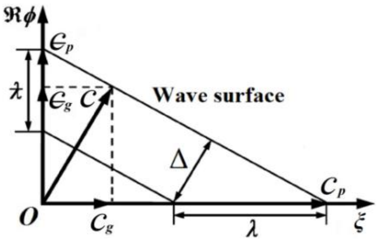

Fig.1Group and phase velocities of photon presented in space $(\xi, \Re\phi)$.

$$c_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = c\left(1 - \frac{c^2 \not{k}^2}{\omega^2 \Re^2}\right)^{1/2}, \qquad c_p = \frac{\omega}{k} = c\left(1 - \frac{c^2 \not{k}^2}{\omega^2 \Re^2}\right)^{-1/2} \tag{16}$$

corresponding to

$$\not{c}_g = \Re \frac{\mathrm{d}\omega}{\mathrm{d}\not{k}} = c\left(1 - \frac{c_g^2}{c^2}\right)^{1/2}, \qquad \not{c}_p = \Re \frac{\omega}{\not{k}} = c\left(1 - \frac{c_g^2}{c^2}\right)^{-1/2} \tag{17}$$

By Eqs.(16) and (17), we can deduce the following additions

$$c_g^2 + \not{c}_g^2 = c^2 , \qquad \frac{1}{c_p^2} + \frac{1}{\not{c}_p^2} = \frac{1}{c^2} \tag{18}$$

being consistent with the geometric representation in generalized space.

Unlike the usual electromagnetic wave theory, Proca pointed out the existence of the third polarized state [5]. Here, we agree with Proca's argument and write the S-wave and P-wave fields as follows

$$\begin{cases} \boldsymbol{E}_{\mathrm{S}} = \dfrac{i\omega}{c} \boldsymbol{A}_{\mathrm{S}}, & \boldsymbol{B}_{\mathrm{S}} = i\boldsymbol{k} \times \boldsymbol{A}_{\mathrm{S}}, & \not{\boldsymbol{B}}_{\mathrm{S}} = -\dfrac{i\not{k}}{\Re} \boldsymbol{A}_{\mathrm{S}} \\ \boldsymbol{E}_{\mathrm{P}} = \dfrac{ic\not{k}^2}{\omega \Re^2} \boldsymbol{A}_{\mathrm{P}}, & \not{E}_{\mathrm{P}} = -\dfrac{ic\not{k}}{\omega\Re} \boldsymbol{k} \cdot \boldsymbol{A}_{\mathrm{P}}, & \not{\boldsymbol{B}}_{\mathrm{P}} = -\dfrac{i\not{k}}{\Re} \boldsymbol{A}_{\mathrm{P}} \end{cases} \tag{19}$$

with energy flows $\boldsymbol{J}_{\mathrm{S}} = c\boldsymbol{E}_{\mathrm{S}} \times \boldsymbol{B}_{\mathrm{S}}$ and $\boldsymbol{J}_{\mathrm{P}} = c\not{E}_{\mathrm{P}} \not{\boldsymbol{B}}_{\mathrm{P}}$. As a special radiation, P-wave is extremely weak, but as long as existing, it will definitely bring some verifiable effects. For example, when a harmonic oscillator charged Q oscillates at frequency ω and amplitude ℓ , its axial radiation field reads

$$E_{\mathrm{P}} = \frac{Q\ell}{\Re^2 r} \mathrm{e}^{-i\omega t} \tag{20}$$

In which, an identical oscillator charged Q' will be forced to oscillate, and even reach an amplitude

$$l = \frac{3\pi Q c^3 \ell}{Q' \omega^3 \Re^2 r} \tag{21}$$

Hence, by selecting appropriate resonance conditions, there may be a chance to reveal P-wave radiation.

3 Physical status of Hubble law

3.1 Recognition towards Hubble frequency-shift

By now, we have incorporated the force-range $\Re_{\mathrm{e}}$ together with speed constant c into Eq.(2), which requires a 5-D Lorentz transformation with continuous symmetry. According to Noether theorem, this symmetry must determine a conserved quantity with mass-moment dimension, just as translational symmetry or immeasurability of absolute position in space implies momentum conservation. Since it is so, why does relativity not definitely define the conserved quantity? The reason, of course, is that the usual mass-moment does not display its due conservation nature in Minkowski space.

Then, seeing that transformation (7) cannot derive Hubble frequency-shift and define its symmetric conserved quantity, we need to make a choice amongst two alternatives: 1) As a transformation theory working in Minkowski space, Einstein relativity is not suitable for discussing Hubble topics. 2) Lorentz transformation doesn't reach its theoretical boundary, so there is still room for further optimization. The former should be thrown out because Hubble law itself is just a law of electromagnetic propagation. The latter will guide us to develop a more universal transformation to work for relativity.

3.1.1 Finiteness of space depth

In modern cosmology, Hubble redshift is attributed to Doppler's effect caused by space expansion. However, the time-varying property of extension process determines that the most effective way to reveals space expansion is to measure the distance of two co-moving points twice, and then compare the measured results. The premise is to find

a standard ruler that can remain unchanged during expansion, but this violates Einstein's no-memory hypothesis.

Of course, one still has every reason to believe in the existence of standard ruler, and rightfully locate the co-moving point at its midpoint. However, if ask, how will an infinite standard rod (or coordinate axis) co-move in expending space. Considering the geometry of infinite rod, the conclusion seems to be that any position on it can be regarded as its co-moving point. As a result, in any case, no relative expansion appears due to the everywhere co-moving of the standard rod and space. As for how to physically understand the immeasurability of space expansion, the answer has actually been written into the principle of relativity, that is, observation has complete personal or empirical attribute, it doesn't allow any observer to personally detect the extrinsic attributes of his/her own reference frame, just as it doesn't allow Lorentz contraction to be self-detected.

On the other hand, if attributing Hubble redshift to recession, the difference in addition rules of distance and velocity will make Hubble law lose its due invariance, so that under Lorentz transformation, it becomes

$$\frac{u+\upsilon_{\text{rec}}}{1+\beta_u\beta_{\upsilon_{\text{rec}}}}=H\gamma_u(r+\Omega u) \tag{22}$$

Unless $c\to\infty$, the formula seems to allow observer to infer the state of a frame by Hubble redshift, indicating that electromagnetic motion governed by Hubble law has no invariance of Lorentz transformation. However, what does $c\to\infty$ mean? It means that interaction can be transiently transmitted, and this transfer mode, known as action at distance, will inevitably lead to a synchronous response of elastic medium to the source vibration, hereby failing to stimulate traveling waves supported by the vibration phase difference. Hence, in this sense we say, the action at distance is not compatible with traveling wave physics. Especially, in the case of $c\to\infty$, the result that Maxwell equations degenerates to $\nabla\cdot E=\rho_{\text{e}}$ and no electromagnetic wave is yielded, is more illustrative of this point.

Now that space doesn't exhibit measurable expansion, Hubble's discovery can only imply a finite detectable space depth, and the limit value of this depth must be the same with respect to any observation position (required by the principle of relativity), just as the limit speed of interaction propagation is the same to any reference frame.

3.1.2 Position-like Lorentz transformation

If supposing that someone has mastered Hubble law before establishing the constancy of the speed of light, he/she would immediately propose a depth postulate to work with Galileo's principle, that is, the limit depth of space characterized by Hubble radius $\Re$ is a universal constant. Under this limit constraint with endogenous root cause (not from external geometry), Galileo space will be shaped into a pseudo-Galileo one centered on the observer's self, so that in standard configuration, the point $\boldsymbol{r}$ is assigned a 5-D position vector $\tilde{r}^n=\gamma_r(-\Re,\boldsymbol{r},\not{r})$. To maintain the invariance of limit depth, the position vectors defined by two observers with separation distance r_0 must be connected by position-like Lorentz transformation, namely

$$\tilde{r}^n=\alpha^n_l(\gamma_{r_0})\tilde{r}'^l \tag{23}$$

Or written as

$$\begin{cases} r_{//}=\dfrac{r_0+r'_{//}}{1+\beta_{r_0}\beta_{r'_{//}}}, & r_\perp=\dfrac{r'_\perp}{\gamma_{r_0}(1+\beta_{r_0}\beta_{r'_{//}})} \\ \not{r}=\dfrac{\not{r}'}{\gamma_{r_0}(1+\beta_{r_0}\beta_{r'_{//}})}, & \not{r}'=\sqrt{\Re^2-r'^2} \end{cases} \tag{24}$$

This indicates that, if a particle is less than or equal to $\Re$ from an observer, it must also be less than or equal to $\Re$ for any other observation position. Only close enough can restore the usual result $\boldsymbol{r}=\boldsymbol{r}_0+\boldsymbol{r}'$. When the distance between two observers is brought about by their relative motion, i.e. $r_0=ut'$, formula (24) becomes

$$\begin{cases} r_{//}=\dfrac{ut'+r'_{//}}{1+\beta_{ut'}\beta_{r'_{//}}}, & r_\perp=\dfrac{r'_\perp}{\gamma_{ut'}(1+\beta_{ut'}\beta_{r'_{//}})} \\ \not{r}=\dfrac{\not{r}'}{\gamma_{ut'}(1+\beta_{ut'}\beta_{r'_{//}})}, & \not{r}'=\sqrt{\Re^2-r'^2} \end{cases} \tag{25}$$

Hence, the relativity with formula (23) as its working transformation is called spatial relativity, which is valid only

at low speed ($\upsilon << c$).

Since what spatial relativity provides is a transformation of finite space, it must clarify where its effective working boundary lies. This question is always regarded as a serious challenge to the finite model, but here the spatial boundary is identified as the physical horizon that prohibits any observer himself from reaching. It can be referenced that no observer is allowed to catch up with photons (at speed boundary). As for how to view the "outside" of pseudo-Galileo space, one can refer to the status of "superluminal world" in relativity. In brief, the so-called surreal "outside" is neither measurable nor influential, thus being physically meaningless.

3.2 Integration of space and motion

The principle of relativity establishes the mediocrity of space and the equality of inertial frames, but it cannot direct natural motion alone.

3.2.1 Logical basis of space-motion equivalence

Unlike Galileo relativity of completely letting go of motion, Einstein relativity set a limit on it, pointing out that no particle can move faster than c . As a result, when the observer's motion undergoes a change Δu , it will bring a velocity variation to target particle, but the variation efficiency is gradually decreasing, so that there exists

$$\Delta\upsilon = \gamma_{\upsilon}^{-2}\Delta u\,|_{\upsilon\to c} \to 0 \tag{26}$$

The tendency towards zero shows that, the speed limit cannot be reached through only a finite number of transfer operations, physically ruling out the possibility of superluminal motion.

Similarly, as a rule guarantee, transformation (23) provides an appropriate transfer to avoid exceeding the depth limit of space, namely

$$\Delta r = \gamma_{r}^{-2}\Delta r_{0}\,|_{r\to\Re} \to 0 \tag{27}$$

This implies the impossibility of delivering a particle to Hubble radius by a finite number of transfers. In turn, it is to actually require observers not to attempt to set or define positioning coordinates outside the horizon.

As shown in **Table**1, a more targeted analogy between distance and recession convinces us that, even without receding, an illuminant can still radiate Hubble photons. This means that the practicality of Hubble law is to specify how much frequency-shift occurs at a certain distance, rather than how fast the illuminant is receding. That way, Hubble velocity only represents a virtual velocity causing no displacement, and its kinematic significance is that the frequency-shift of a photon from distance r can always be reproduced indistinguishably by a frame receding at speed Hr . Importantly, the indistinguishability of Hubble redshift and Doppler's effect provides a logical basis for us to interpret Hubble constant as velocity equivalent of position.

Table1.The equivalent performances of distance to recession.

Target	Distance	Recession
Absolute value	Unmeasurable	Unmeasurable
Limit	$\Re$	c
Frequency-shift law	$1+Z=\sqrt{\frac{1+\beta_r}{1-\beta_r}}$	$1+Z=\sqrt{\frac{1+\beta_{\upsilon_{\rm rec}}}{1-\beta_{\upsilon_{\rm rec}}}}$

3.2.2 Requirement for transformation

From the practical effect, neither Einstein relativity nor spatial relativity seems to be able to achieve a comprehensive description of electromagnetic motion. For example, the former cannot derive Hubble frequency-shift, and the latter doesn't restrict motion velocity. Only by reshaping space and optimizing Lorentz transformation can get rid of the dilemma, but requires:

1) As a stage for all things in nature to perform their motion, the reshaped space must be universally accessible, completely describable and non-fragmented, especially simple in form and explicit in picture. In particular, it should be a solution of Einstein equation, but not derived from the reconciliation of various uncertain factors.

2) The optimized transformation must keep physical laws including Hubble law invariant, so as to avoid

inferring the position or motion state of a frame through Hubble frequency-shift. To this end, it is necessary to reevaluate the significance of space and motion, and further unify them into a concept of full-velocity.

3.3 Materiality of space

Lorentz transformation fuses spacetime with motion, but only the gravitational mechanism of matter shaping space can help us develop a universal transformation form with cosmological significance.

3.3.1 Potential field and matter

In measured space, we solve Laplace equation $\nabla_r^2\Phi = 4\pi G\rho_m$, and then have gravitational potential

$$\Phi(r) = -\varepsilon\rho_m\left(1-\frac{r^2}{2\Re_\mathrm{g}^2}\right)c^2 , \qquad \varepsilon = \frac{4\pi G\Re_\mathrm{g}^2}{3c^2} \tag{28}$$

ρ_m is mass density. Taking the horizon as zero potential $\Phi|_{r\to\Re}=0$ assigns the gravitational force-range

$$\Re_\mathrm{g} = \frac{\Re}{\sqrt{2}} \tag{29}$$

As this force-range approaches infinity, the potential $\Phi\to\infty$, leading to Seelinger paradox. If trying to interpret relativistic energy mc^2 as a "potential energy" of mass-charge m in space, it is necessary to make a dark energy compensation $\widehat{\rho}$ for the potential field until the background potential Φ_b has a desired background value,

$$\Phi_\mathrm{b} = -\varepsilon(\rho_m c^2 + \widehat{\rho}) = c^2 \tag{30}$$

Not only that, based on the thermodynamic analysis of matter system in a given volume V, one can find that due to no macroscopic energy exchanging with its surrounding environment, there must be an equilibrium equation for volume fluctuation δV, that is, $\rho_m c^2\delta V + p_\mathrm{rel}\delta V = 0$. Then, if assuming that relativistic negative pressure p_rel against gravitation is generated from dark energy, then we have

$$p_\mathrm{rel} = \frac{\widehat{\rho}}{3} = -\rho_m c^2 , \qquad \rho_m\varepsilon = \frac{1}{2} \tag{31}$$

On the other hand, the equivalence of distance to recession allows us to treat the current world as expanding from a primitive extreme state with Planck density $\rho_\mathrm{P}(=c^5/\hbar G^2)$. This extreme state is considered to be dominated by Yukawa interaction with an effective force-range

$$\Re_\mathrm{s} = \frac{\hbar}{m_\pi c} \tag{32}$$

where, m_π is π-meson mass. So, by Eq.(31) and the conservation relation $\rho_\mathrm{P}\Re_\mathrm{s}^3 = \rho_m\Re_\mathrm{g}^3$, we get

$$\Re_\mathrm{g} = \frac{8\pi\hbar^2}{3Gm_\pi^3} , \qquad \rho_m = \frac{27Gm_\pi^6 c^2}{512\pi^3\hbar^4} \tag{33}$$

With mass m_π as a link, it highlights the decisive role of material density in shaping space.

3.3.2 Cosmic background radiation temperature

Now, a predictable situation that photons shining toward the horizon are not reflected, suggests us to view the whole space as a large blackbody cavity, so that the total internal energy and entropy of cosmic background radiation in volume V obey thermodynamic relation

$$U_\gamma^{3/4} = \frac{3}{8}\left(\frac{4c}{\sigma V}\right)^{1/4} S_\gamma \tag{34}$$

σ is Stefan constant. If suppose that the emission ratio defined by Rydberg constant R_∞ of atomic radiation is of general significance, and even represents the proportion of the background radiation u_γ in space matter, we have

$$\frac{u_\gamma}{\rho_m c^2} = \frac{\hbar\omega_\infty}{m_\mathrm{e}c^2} = \frac{\alpha^2}{2} , \qquad \omega_\infty = 2\pi R_\infty c \tag{35}$$

m_e denotes electron mass, and α the fine structure constant. Then, with the help of (33) and (35), we can derive the background radiation temperature

$$T_{\mathrm{b}}=\frac{\partial U_{\gamma}}{\partial S_{\gamma}}=\left(\frac{27Gm_{\pi}^{6}c^{5}\alpha^{2}}{4096\pi^{3}\hbar^{4}\sigma}\right)^{1/4}=\begin{cases}2.667\mathrm{K}, & m_{\pi}=m_{\pi0}\\ 2.734\mathrm{K}, & m_{\pi}=\overline{m}_{\pi}\\ 2.805\mathrm{K}, & m_{\pi}=m_{\pi\pm}\end{cases} \tag{36}$$

Where, $m_{\pi0}(=2.406\times10^{-28}\mathrm{kg})$ and $m_{\pi\pm}(=2.488\times10^{-28}\mathrm{kg})$ are the masses of mesons $(\pi^{0},\pi^{\pm})$, whose weighted average $\overline{m}_{\pi}$ is defined by

$$\int_{0}^{\infty} r\mathrm{e}^{-\frac{\overline{m}_{\pi}c}{\hbar}r}\mathrm{d}r=\frac{1}{2}\int_{0}^{\infty} r\mathrm{e}^{-\frac{m_{\pi0}c}{\hbar}r}\mathrm{d}r+\frac{1}{2}\int_{0}^{\infty} r\mathrm{e}^{-\frac{m_{\pi\pm}c}{\hbar}r}\mathrm{d}r \tag{37}$$

giving

$$\overline{m}_{\pi}=\sqrt{\frac{2m_{\pi0}^{2}m_{\pi\pm}^{2}}{m_{\pi0}^{2}+m_{\pi\pm}^{2}}}=2.446\times10^{-28}\mathrm{kg} \tag{38}$$

When m_{π} is counted as the weighted average mass of mesons, the result is in agreement with the measured background temperature $T_{\mathrm{b}}=2.726\pm0.010\mathrm{K}$ [6].

4 Full-velocity relativity

4.1 Full-velocity Lorentz transformation

4.1.1 Velocity equivalent of position

In classical physics, inertial motion refers to a free particle persisting in its original velocity (frequency) in a straight line. Alternatively, it can be rephrased as that the velocity of free particle relative to a target plus that of the target remains unchanged. As such, if Hubble particle cannot maintain its motion, is there any other invariant left? The invariant left is full-velocity, which (as the rephrased above) represents the sum of the displacement velocity of a particle passing through a given point $\boldsymbol{r}$ and Hubble velocity of that point, namely

$$\boldsymbol{\upsilon}_{\mathrm{F}}=\boldsymbol{\upsilon}_{\mathrm{D}}+H\boldsymbol{r} \tag{39}$$

Physically, the redefined inertia not only integrates motion and space into a full-velocity concept, but also allows for the interpretation of Hubble constant H as velocity equivalent of position. In particular, recognition of this equivalent will require us to elevate the physical status of Hubble law.

4.1.2 Restatement of Einstein's postulates

Given the reality that free particles cannot persist in their original velocity (frequency) in confined space, we make an adjustment to the definition of inertial frame, that is, a frame can work as an inertial one only if it moves at a constant full-velocity relative to a free particle instead of the usual velocity.

Hereby, based on the redefinition of inertial frame by full-velocity, we restate Einstein's postulates as:

The principle of relativity *Physical laws are the same in all full-velocity inertial frames, or, no physical experiment can reveal the absolute motion or position of a reference frame.*

Constancy of the full-velocity limit of light *The full-velocity limit of light has the same value* c *in all full-velocity inertial frames.*

The first highlights that all full-velocity inertial frames are equivalent in physical description. The second provides an endogenous constraint mechanism for reshaping space. This reshaped space, called pseudo-Minkowski space, is not a superior manifold beyond reality, but a "living stage" for performing relativistic motion

Obeying two postulates, any occurrence in nature is observable in principle rather than physically isolatable, and the transformation that keeps physical laws invariant becomes full-velocity Lorentz transformation

$$\tilde{r}^{n}=\alpha_{l}^{n}(\gamma_{u_{\mathrm{F}}})\tilde{r}'^{l}, \qquad \tilde{r}'^{l}=(-c\tilde{t}',\gamma_{r'}\boldsymbol{r}',\Re\tilde{\phi}') \tag{40}$$

For Hubble particle with full-velocity υ_{F}, we clarify

$$\tilde{t}=\frac{\gamma_{r}r}{\upsilon_{\mathrm{F}}}, \qquad \tilde{\phi}=\sqrt{1-\beta_{\upsilon_{\mathrm{F}}}^{2}}H\tilde{t} \tag{41}$$

and then have a 5-D Hubble relation

$$\dot{\tilde{r}}^n = (-c, \boldsymbol{v}_{\mathrm{F}}, \cancel{\boldsymbol{v}}_{\mathrm{F}}) = H(-\Re, \boldsymbol{r}_{\mathrm{F}}, \cancel{\boldsymbol{r}}_{\mathrm{F}}) , \qquad \boldsymbol{r}_{\mathrm{F}} = \Omega \boldsymbol{v}_{\mathrm{F}} \tag{42}$$

$\boldsymbol{r}_{\mathrm{F}}$ is the finally-arrival position of particle, called full-position. As $\tilde{t}' = \Omega$ and $u_{\mathrm{F}} = Hr_0$, transformation (40) reduces to (23), so that replacing the measured distance with the backtracking-time $\tau' = r'/c$ yields

$$\tau = \frac{\tau_0 + \tau'}{1 + \beta_{\tau_0}\beta_{\tau'}} = \begin{cases} \tau_0 + \tau', & \tau_0 << \Omega, \quad \tau' << \Omega \\ \Omega, & \tau_0 = \Omega \ \text{ or } \ \tau' = \Omega \end{cases} \tag{43}$$

This addition tells us that, for a past event, provided its backtracking time τ' to a previous observer does not exceed Ω, then the total backtracking time τ in the view of any successor (at any time or position) can never exceed Ω. That way, although time is constantly losing, the maximum historical span it left is always Ω, thus calibrating an absolute starting point of time for the relativistic world, which is the same for all observers.

Furthermore, by Eq.(40), it is easy to derive

$$\begin{cases} v_{\mathrm{F}//} = \dfrac{u_{\mathrm{F}} + v'_{\mathrm{F}//}}{1 + \beta_{u_{\mathrm{F}}}\beta_{v'_{\mathrm{F}//}}}, & v_{\mathrm{F}\perp} = \dfrac{v'_{\mathrm{F}\perp}}{\gamma_{u_{\mathrm{F}}}(1 + \beta_{u_{\mathrm{F}}}\beta_{v'_{\mathrm{F}//}})} \\ \cancel{v}_{\mathrm{F}} = \dfrac{\cancel{v}'_{\mathrm{F}}}{\gamma_{u_{\mathrm{F}}}(1 + \beta_{u_{\mathrm{F}}}\beta_{v'_{\mathrm{F}//}})}, & \cancel{v}'_{\mathrm{F}} = \sqrt{c^2 - v'^2_{\mathrm{F}}} \end{cases} \tag{44}$$

Or decomposed into two-component form

$$\begin{cases} \begin{bmatrix} v_{\mathrm{D}//} \\ r_{//} \end{bmatrix} = \Theta \begin{bmatrix} u_{\mathrm{D}} + v'_{\mathrm{D}//} \\ r_0 + r'_{//} \end{bmatrix}, & \begin{bmatrix} v_{\mathrm{D}\perp} \\ r_{\perp} \end{bmatrix} = \Theta \gamma^{-1}_{\left[\begin{smallmatrix} u_{\mathrm{F}} \\ r_{0\mathrm{F}} \end{smallmatrix}\right]} \begin{bmatrix} v'_{\mathrm{D}\perp} \\ r'_{\perp} \end{bmatrix} \\ \begin{bmatrix} \cancel{v}_{\mathrm{F}} \\ \cancel{r}_{\mathrm{F}} \end{bmatrix} = \Theta \gamma^{-1}_{\left[\begin{smallmatrix} u_{\mathrm{F}} \\ r_{0\mathrm{F}} \end{smallmatrix}\right]} \begin{bmatrix} \cancel{v}'_{\mathrm{F}} \\ \cancel{r}'_{\mathrm{F}} \end{bmatrix}, & \begin{cases} \cancel{v}'_{\mathrm{F}} = \sqrt{c^2 - v'^2_{\mathrm{F}}} \\ \cancel{r}'_{\mathrm{F}} = \sqrt{\Re^2 - r'^2_{\mathrm{F}}} \end{cases} \end{cases}, \qquad \Theta = \frac{1}{1 + \beta_{\left[\begin{smallmatrix} u_{\mathrm{F}} \\ r_{0\mathrm{F}} \end{smallmatrix}\right]} \beta_{\left[\begin{smallmatrix} v'_{\mathrm{F}//} \\ r'_{\mathrm{F}//} \end{smallmatrix}\right]}} \tag{45}$$

In the case of $r << \Re$ ($v << c$), the developed will naturally reduce to the velocity-like (position-like) Lorentz transformation, and even to Galileo transformation when both are met simultaneously.

4.1.3 Transformation symmetry

After incorporating full-velocity motion, Lorentz transformation intuitively displays a peer-to-peer symmetry of position and velocity, and this symmetry must lead to two matching conservation laws. To this end, we introduce mass-inertia $\bar{I}$ and rewrite Eq.(42) as

$$\bar{P}^n = H\bar{M}^n , \qquad \bar{M}^n = \left(-\Re^{-1}I, m_0\gamma_{r_{\mathrm{F}}}\vec{r}_{\mathrm{F}}\right), \qquad \bar{I} = m_0\gamma_{r_{\mathrm{F}}}\Re^2 \tag{46}$$

It interprets full-velocity energy-momentum $\bar{P}^n$ as the equivalent of full-position mass-moment $\bar{M}^n$. Consequently, evaluating the norm of two quantities yields

$$\bar{E}^2 = \bar{P}^2c^2 + m_0^2c^4 , \qquad \bar{I}^2 = \bar{M}^2c^2 + m_0^2\Re^4 \tag{47}$$

Behind which is the full-velocity (position) Lorentz transformation, whose symmetry determines the conservation of $\bar{M}^n$ ($\bar{P}^n$). As $\Re \to \infty$, $\bar{M}^n$ will lose physical significance due to its tending to infinity. This is why Einstein relativity cannot definitely define Noether conservation law based on the symmetry of Lorentz transformation.

If applying the optimized transformation to a photon ω_0 from distance r, we derive a stricter redshift form

$$1 + Z_{\mathrm{str}} = \frac{\lambda}{\lambda_0} = \frac{\beta_0}{\gamma_r(\beta_0 - \beta_r)}, \qquad \beta_0 = \sqrt{1 - \frac{H^2}{\omega_0^2}} \tag{48}$$

Where, r is required to be less than the infinite redshift distance of the photon, i.e. $r \le \Re\beta_0$. Usually, due to $\omega_0 >> H$, it actually has no difference from the redshift relation in **Table**1.

4.2 Consequences of transformation

4.2.1 Relativity of simultaneity

Consider a pair of simultaneous events $P_1(0,0)$ and $P_2(0 + \mathrm{d}\tilde{r}', 0 + \mathrm{d}\tilde{t}')$ in frame S', we can write its time interval in S far from S' with distance r_0 as

$$\mathrm{d}\tilde{t} = \frac{\gamma_r r_0}{c\Re}\mathrm{d}\tilde{r}' \ne \mathrm{d}\tilde{t}' = 0 \tag{49}$$

This non-zero interval tells us, two simultaneous events seen by one observer don't necessarily appear to occur

simultaneously to other observer at different position.

4.2.2 Time dilation and length contraction

To interpret that time slows down at a distance, let's imagine a light clock of ticking away the time by light-pulse bouncing back and forth with length $L_0 (<< \Re)$. So, in the view of local observer, the duration of a round trip is T_0, while, for an observer at distance r from the clock (in perpendicular), the light will travel its round trip at speed c_{D} in time interval

$$T = \frac{2L_0}{c_{\mathrm{D}}} = \gamma_r T_0 , \qquad c_{\mathrm{D}} = \frac{c_g}{\gamma_r} \tag{50}$$

showing time dilation responsible for Hubble reshift. Anyway, to avoid the horizon being exceeded, it must slow down the far-distance motion, and only time dilation can fill the time gap between two observation positions.

However, if viewing the light clock longitudinally at distance r, its round time will become

$$T = \frac{L}{c_{\mathrm{D}_+}} + \frac{L}{c_{\mathrm{D}_-}} , \qquad c_{\mathrm{D}_\pm} = \frac{c_g \pm Hr}{1 \pm \beta_r \beta_{c_g}} \mp Hr \tag{51}$$

L is the pulse distance. Then, with the help of Eq.(50), we deduce length contraction

$$L = \gamma_r^{-1} L_0 \tag{52}$$

It is the contraction effect caused by distance that rules out the possibility of exceeding the horizon.

4.2.3 Strong causality

As a physical prerequisite for supporting the existence of traveling waves, the finite propagation speed of interactions will inevitably have a restrictive effect on space. To illustrate this, we write the time interval recorded by observer S for causal events P_1 and P_2 (connected by velocity $v'_{\mathrm{inf}} (\le c)$), namely

$$\mathrm{d}\tilde{t} = \gamma_r \left(1 - \frac{r v'_{\mathrm{inf}}}{\Re c} \right) \mathrm{d}\tilde{t}' \ge 0 \tag{53}$$

The result indicates that, to maintain time sequence and avoid space being segmented must require $r \le \Re$. Only so can causal events be ensured to occur within a limited scope, thus giving the causal chain a logically traceable finite length. After all, allowing the infinite space and time span of causal events defined in Minkowski space has no logical practical significance

Hereby, we reiterate three absolute meanings of strong causality:

a) *There is no reference system in which the causal sequence of physical events can be reversed or broken.*

b) *No event in nature can be absolutely isolated by the horizon, so that it can never be observed or described.*

c) *The cause of any given event can always be tracked within limited time and space ranges.*

The first is from a natural requirement of logic—no effect can precede its cause or no causal chain can be cut off, the second ensures the completeness of physical description—no effect (cause) has no cause (effect), and the third makes any causal reasoning operable in principle—no causal pair has infinite time and space span. Only when the three are met simultaneously can a relativistic transformation be practically executable. Undoubtedly, full-velocity Lorentz transformation has withstood the rigorous test from strong causality, which not only ensures the orderliness, continuity and traceability of logic chain, but also achieves completeness of physical description, having stronger operability and explanatory function.

4.3 Consequences of full-velocity transformation

4.3.1 Measured duration and depth

For an event pair of $(\Delta\tilde{r}''=0, \Delta\tilde{t}''=T_0)$ occurred at the origin of frame S'' with velocity u relative to S', its two intervals in S at distance r_0 far from S' read

$$\Delta\tilde{r} = \frac{(u + Hr_0)T_0}{\sqrt{(1-\beta_u^2)(1-\beta_{r_0}^2)}} , \qquad \Delta\tilde{t} = \frac{(1+\beta_u \beta_{r_0})T_0}{\sqrt{(1-\beta_u^2)(1-\beta_{r_0}^2)}} \tag{54}$$

Then, by Eq.(40) we have $\Delta r = \Delta\tilde{r} / \sqrt{1+\beta_{\Delta\tilde{r}}^2}$, determining a duration

$$T=\frac{\Delta r}{\Delta\tilde{r}}\Delta\tilde{t}=\frac{(1+\beta_u\beta_{r_0})T_0}{\sqrt{(1-\beta_u^2)(1-\beta_{r_0}^2)+(\beta_u+\beta_{r_0})^2(HT_0)^2}}$$
$$=\begin{cases}\gamma_u T_0, & T_0<<\Omega,\quad r_0=0\\ \gamma_{r_0}T_0, & T_0<<\Omega,\quad u=0\\ \Omega, & T_0\to\Omega\ \text{ or }\ u\to c\ \text{ or }\ r_0\to\Re\end{cases}\tag{55}$$

As $T_0<<\Omega$ and $r_0=0$ ($u=0$), it gives usual (spatial) dilation. Once any of $T_0\to\Omega$, $u\to c$ and $r_0\to\Re$ is met, it yields the maximum duration, meeting the requirement that time span be less than or equal to Ω.

Similarly, to illustrate the contraction of measured distance, let's write the generalized value of a longitudinal length L_0 in frame S'' as $\tilde{r}''=\gamma_{L_0}L_0$. This value in will undergo a Lorentz contraction

$$\tilde{r}=\gamma_{L_0}L_0\sqrt{1-\frac{(\beta_u+\beta_{r_0})^2}{(1+\beta_u\beta_{r_0})^2}}\tag{56}$$

corresponding to a measured length

$$L=\frac{\tilde{r}}{\sqrt{1+\beta_{\tilde{r}}^2}}=\frac{L_0\sqrt{(1-\beta_u^2)(1-\beta_{r_0}^2)}}{\sqrt{(1+\beta_u\beta_{r_0})^2-(\beta_u+\beta_{r_0})^2\beta_{L_0}^2}}$$
$$=\begin{cases}L_0\sqrt{1-\beta_u^2}, & L_0<<\Re,\quad r_0=0\\ L_0\sqrt{1-\beta_{r_0}^2}, & L_0<<\Re,\quad u=0\\ \Re, & L_0\to\Re\end{cases}\tag{57}$$

When $L_0<<\Re$ and $r_0=0$ ($u=0$), it reduces to the usual (spatial) Lorentz contraction. Once $L_0\to\Re$, it gives the maximum space depth $\Re$, implying that all observers have a common horizon characterized by Hubble radius.

In general, full-velocity Lorentz transformation has following properties:

1) Works in pseudo-Minkowski space and achieves the full coverage of it.

2) Depends on both observation position and reference velocity.

3) Ensures the causal chain unbroken and the physical description complete.

4) Has the continuous symmetry corresponding to mass-moment conservation.

5) Reduces to the velocity (position)-like one for local range (low speed), and even to the Galileo's for both.

4.3.2 Spacetime diagram and historization

According to full-velocity relativity, although time always evenly and irreversible advances towards the future in a mathematical sense, what it leaves behind is a traceable limited history. **Fig**.2 shows a spacetime diagram with past-time T and mathematical future-time t as the past-future axe, which is divided into two parts by the present line. The upper depicts an absolute future closed in space but open in time, the lower focuses on an absolute past closed in both space and time.

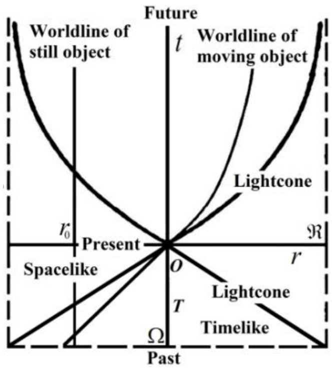

Fig.2 By the past and future lightcones, pseudo-Minkowski space is separated into spacelike and timelike intervals with respect to event O at the origin.

In pseudo-Minkowski space, the motion equation of free particles is $\dot{r}+Hr=\upsilon_F$, whose solution will trace out a curved trajectory in the upper

$$r=\upsilon_F\Omega(1-e^{-Ht})\tag{58}$$

but a straight line in the lower $r=\upsilon_F T$, collectively called world line. As $\upsilon_F\to c$, past-time T reads backtracking time τ, and in such event the world line tends to light-cone, within which all events that may be influenced by (can influence) event O constitute its absolute future (past). However, with the continuous advancement of present line to the future, all occurrences will ultimately be destined to fall into the downward light-cone, and even

deposit at the cone bottom with a depth Ω, marked as the absolute starting point of time.

Once mathematical future-time t is irreversibly converted into the traceable past, its historical span reads

$$T=\Omega(1-\mathrm{e}^{-Ht}) \tag{59}$$

called historization or backtracking effect. It is this historization that destroys the inversion symmetry of time, only when $t<<\Omega$, can it leave a traceable history of equal span, that is $T=t$.

Compared with usual relativity, full-velocity relativity not only constructs a compact spatial manifold that is compatible with relativistic cosmology, but also develops the matching working transformation that can cater to the perfect causal law, thus exhibiting stronger inclusiveness and explanatory functions. **Table**2 shows the comparison of several relativistic forms on related issues.

Table2 Comparison of different relativistic forms.

Relativity	Galileo relativity	Spatial relativity	Einstein relativity	Full-velocity relativity
Space	Galileo's	Pseudo-Galileo's	Minkowski's	Pseudo-Minkowski's
Transformation	Galileo's	Position-type	Velocity-type	Full-velocity-type
Space domain	$r<<\Re$	$0\le r\le\Re$	$r<<\Re$	$0\le r\le\Re$
Velocity domain	$\upsilon<<c$	$\upsilon<<c$	$0\le\upsilon\le c$	$0\le\upsilon\le c$
Time starting point	No	No	No	Yes
Electromagnetism	$\nabla\cdot E=\rho$	$\vec{\nabla}\cdot\vec{E}=\rho$	$F^{nl}_{,l}=j^n/c$	$F^{nl}_{;l}=j^n/c$
Frequency-shift law	$1+Z=1$	$1+Z=1$	$1+Z=1$	$1+Z=\gamma_r(1+\beta_r)$

5 General relativity analysis of pseudo-Minkowski space

5.1 Pseudo-Minkowski universe

5.1.1 Pseudo-Minkowski metric

On issues related to cosmology, full-velocity relativity always upholds a fundamental view that the current universe as a whole is not the stack of all the concepts, but an absolute closed collection of all detectable contents. This absolute closure determines that the universe can only explain itself on its own, but cannot rely on any extrinsic mechanism, and even introducing geometric constraint beyond the perceptible dimensions is not allowed.

In pseudo-Minkowski space, we convert the form of $0=-c^2+\upsilon_{\mathrm{F}}^2+\bar{\upsilon}_{\mathrm{F}}^2$ into an expression of invariant interval

$$\mathrm{d}s^2=-c^2\mathrm{d}t^2+(\mathrm{d}r+rH\mathrm{d}t)^2+r^2\mathrm{d}\theta^2+r^2\sin^2\theta\mathrm{d}\varphi^2+\mathrm{d}\xi^2 \tag{60}$$

and so have pseudo-Minkowski metric

$$\bar{\eta}_{nl}=\begin{bmatrix}-c^2(1-\beta_r^2) & Hr & 0 & 0 & 0\\ Hr & 1 & 0 & 0 & 0\\ 0 & 0 & r^2 & 0 & 0\\ 0 & 0 & 0 & r^2\sin^2\theta & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{61}$$

This metric satisfies the cosmological principle, which interprets the infinite redshift surface as its singularity at Hubble radius. When $r<<\Re$ or $\Re\to\infty$ (i.e. $\rho_m\to 0$), it reduces to Minkowski metric, reflecting the role of matter as the subject of motion in shaping relativistic space. So in this sense, we say that space and time are integrated, and closely related to matter and its motion, embodying the high unity of spacetime, matter and motion.

5.1.2 Cosmological solution

In the expansion model, our universe is viewed as a subspace of high-dimensional space with an imperceptible redundant degree of freedom, so that it becomes a non-purely autonomous evolutionary system under external geometric constraints. However, a definition of the universe that encompasses everything is actually a default on its absolute isolation, which in turn determines that any model attempting to explain this all-encompassing system must rely on inherent mechanism rather than seeking extrinsic reason. Hereby, we propose the pseudo-Minkowski universe constrained by the specification of endogenous full-velocity limit, and write Einstein field equation to govern its dynamical behavior (including the Λ term), namely

$$R_n^l - \frac{1}{2}R\delta_n^l + \Lambda\delta_n^l = -\frac{8\pi G}{c^4}\overline{T}_n^l \tag{62}$$

with a stress-energy tensor

$$\overline{T}_n^l = (\rho_m c^2 + p_{\mathrm{rel}})U_n U^l + p_{\mathrm{rel}}\delta_n^l\,, \qquad U^l = \frac{1}{\sqrt{-g_{00}}}(-1,0,0,0,1) \tag{63}$$

U^l is the 5-velocity of still fluid element.

By metric (61), we find curvature tensor

$$R_n^l = -3\delta_n^l\,, \quad n,l \neq 4 \tag{64}$$

All other R_n^l are identically zero, reflecting the zero influence of stress-energy on the extra-component. Then, the equation says that the non-zero terms should be

$$\frac{3}{\Re^2}\delta_n^l + \Lambda\delta_n^l = \frac{3}{\Re_{\mathrm{g}}^2}\delta_n^l \;, \qquad \Lambda = \frac{3}{\Re^2} \tag{65}$$

Only with such values can the pseudo-Minkowski universe be precisely in dynamic equilibrium.

With the obtained in hand, it is easy to give the natural constant expressions of some relevant parameters (see **Table**3). Of course, if necessary, one can take background temperature T_{b} (the only accurately measured quantity at present) as a characteristic indicator of the universe, and use it to express other parameters, for example

$$H = \sqrt{\frac{32\pi G\sigma}{3\alpha^2 c^3}}T_{\mathrm{b}}^2\Big|_{T_{\mathrm{b}}=2.726\mathrm{K}} = 68.17\mathrm{km\cdot s^{-1}\cdot Mpc^{-1}} \tag{66}$$

The assigned value is consistent with the observed result $67.80 \pm 0.77\mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$ [7]. Importantly, the obtained shows, despite being in a static state, the pseudo-Minkowski universe can still reproduce the effective consequences of expansion model, while avoiding creation singularity and horizon difficulty.

Table3.Relatant cosmic parameters expressed by natural constants ($\overline{m}_\pi = 2.446\times10^{-28}\mathrm{kg}$).

Parameter	Hubble radius	Hubble constant	Hubble time	Mass density	Cosmic temperature
Expression	$\Re = \frac{8\sqrt{2}\pi\hbar^2}{3G\overline{m}_\pi^3}$	$H = \frac{3G\overline{m}_\pi^3 c}{8\sqrt{2}\pi\hbar^2}$	$\Omega = \frac{8\sqrt{2}\pi\hbar^2}{3G\overline{m}_\pi^3 c}$	$\rho_m = \frac{27G\overline{m}_\pi^6 c^2}{512\pi^3\hbar^4}$	$T_{\mathrm{b}} = \left(\frac{27G\overline{m}_\pi^6 c^5\alpha^2}{4096\pi^3\hbar^4\sigma}\right)^{1/4}$
Value	$1.35\times10^{26}\mathrm{m}$	$68.56\mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$	$1.43\times10^{10}\mathrm{a}$	$1.77\times10^{-26}\mathrm{kg\cdot m^{-3}}$	2.734K
Parameter	Gravitational-range	Graviton mass	Electromagnetic-range	Photon mass	Cosmic constant
Expression	$\Re_{\mathrm{g}} = \frac{8\pi\hbar^2}{3G\overline{m}_\pi^3}$	$m_{\mathrm{g}} = \frac{3G\overline{m}_\pi^3}{8\pi\hbar c}$	$\Re_{\mathrm{e}} = \frac{8\sqrt{2}\pi\hbar^2}{3G\overline{m}_\pi^3}$	$m_\gamma = \frac{3G\overline{m}_\pi^3}{8\sqrt{2}\pi\hbar c}$	$\Lambda = \frac{27G^2\overline{m}_\pi^6}{128\pi^2\hbar^4}$
Value	$9.54\times10^{25}\mathrm{m}$	$3.69\times10^{-69}\mathrm{kg}$	$1.35\times10^{26}\mathrm{m}$	$2.61\times10^{-69}\mathrm{kg}$	$1.65\times10^{-52}\mathrm{m^{-2}}$

5.1.3 Geodesic equation

Moving in pseudo-Minkowski space, Hubble particles follow the geodesic equation

$$\frac{\mathrm{d}^2 r^n}{\mathrm{d}\phi^2} + \Gamma_{lk}^n\frac{\mathrm{d}r^l}{\mathrm{d}\phi}\frac{\mathrm{d}r^k}{\mathrm{d}\phi} = 0\,, \qquad \Gamma_{lk}^n = \frac{1}{2}g^{no}(g_{lo,k} + g_{ok,l} - g_{lk,o}) \tag{67}$$

Γ_{lk}^n denotes the affine connection. For a photon ω_0 departing from the origin, it yields

$$\dot{r} + Hr = c\beta_0 \tag{68}$$

Of which, the solution reads

$$\begin{cases} r(t) = \Re\beta_0(1-\mathrm{e}^{-Ht}) \\ c_{\mathrm{D}}(t) = c\beta_0\mathrm{e}^{-Ht} \end{cases} \tag{69}$$

This expression shows that Hubble photons can always approach at a decay speed, but never cross the horizon.

Meanwhile, for a photon ω_0 from distance r_0, the geodesic equation becomes

$$\dot{r} + Hr = \frac{Hr_0 - c\beta_0}{1-\beta_{r_0}\beta_0} \tag{70}$$

Of which, the solution reads

$$\begin{cases} r(t)=\dfrac{r_0-\Re\beta_0}{1-\beta_{r_0}\beta_0}(1-\mathrm{e}^{-Ht})+r_0\mathrm{e}^{-Ht}\Big|_{\substack{\omega_0\to\infty\\ r_0\sim\Re}}^{t\to\infty}\to-\Re \\ c_{\mathrm{D}}(t)=-\dfrac{c\beta_0(1-\beta_{r_0}^2)}{1-\beta_{r_0}\beta_0}\mathrm{e}^{-Ht}\Big|_{\substack{\omega_0\to\infty\\ r_0\sim\Re}}^{t=0}\to-2c \end{cases} \tag{71}$$

The first tells us, even if coming from a position close to the horizon, an ultra-high energy photon will not lose all its energy after reaching the origin, and still has the opportunity to fly across the whole space. The second gives the separation velocity of target photon from the horizon $|c_{\mathrm{D}}(0)|\sim 2c$, but this does not bring about contradiction. In fact, the full-velocity of any moving body measured by observer at any position can never be greater than c. If notice that after reaching the origin, the displacement velocity of target photon reads $c_{\mathrm{D}}=c_{\mathrm{F}}$, then by Eq.(71) we can deduce redshift relation (48). In this way, the transformation result and geodesic motion achieves mutual corroboration, illustrating the cosmological significance of full-velocity Lorentz transformation.

5.2 Gravitational field

5.2.1 Weak field approximation

In the case of weak field, the spatial metric and stress-energy tensor can be expressed respectively as

$$g_{nl}=\bar{\eta}_{nl}+h_{nl}\,,\qquad T_{nl}=\bar{T}_{nl}+T'_{nl} \tag{72}$$

where, $|h_{nl}|<<|g_{nl}|$ and $T'_{nl}<<\bar{T}_{nl}$. In standard configuration, we can separate a linear part from the field equation

$$\bar{h}_{nl,o}^{\;,o}=-\frac{16\pi G}{c^4}T'_{nl}\,,\qquad \bar{h}_{nl}=h_{nl}-\frac{1}{2}\bar{\eta}_{nl}h \tag{73}$$

For the gravitational wave propagating in pseudo-Minkowski space ($T'_{nl}=0$), it becomes

$$h_{nl,o}^{\;,o}=0 \tag{74}$$

By which, we can get the radial frequent dependence of massive graviton $\omega^2=(ck_r+\beta_r\omega)^2+c^2\not{k}^2/\Re_{\mathrm{g}}^2$, indicating that graviton still obeys Hubble law.

5.2.2 Spherical external solution

To get the static gravitational field around an isolated mass point M in pseudo-Minkowski space with non-zero mass density, we look for a spherically symmetric metric

$$\begin{aligned}\mathrm{d}s^2&=-c^2W_1^2(r)(1-\beta_r^2)\mathrm{d}t^2+2Hr\mathrm{d}r\mathrm{d}t+W_2^2(r)\mathrm{d}r^2\\&\quad+W_3^2(r)r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2)+\mathrm{d}\xi^2\end{aligned} \tag{75}$$

Where, W_1, W_2, W_3 are functions of radius r, and due to r being calibrated by $\Re$, making transformation $r\to r'=W_3r$ or setting $W_3=1$ is not allowed. Then, it is required that these three functions be determined so as to satisfy Einstein equation. By the standard analysis, we deduce

$$W_1=W_2^{-1}=W_3^{-1}=\mathrm{e}^{-\vartheta(r)}\,,\qquad T_n^l=p_{\mathrm{rel}}\left(\frac{1+\mathrm{e}^{-2\vartheta(r)}}{2}\right)\delta_n^l \tag{76}$$

with the exponential parameter reading

$$\vartheta(r)=\frac{GM}{c^2}\left(\frac{1}{r}-\frac{\ln(1+Z)}{\Re}\right) \tag{77}$$

As $M\to 0$, the stress-energy tensor tends to the background value $T_n^l\to p_{\mathrm{rel}}\delta_n^l$. Hence, we get the analytic solution

$$\begin{aligned}\mathrm{d}s^2&=-c^2\mathrm{e}^{-2\vartheta(r)}(1-\beta_r^2)\mathrm{d}t^2+2Hr\mathrm{d}r\mathrm{d}t\\&\quad+\mathrm{e}^{2\vartheta(r)}[\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2)]+\mathrm{d}\xi^2\end{aligned} \tag{78}$$

This solution has a physical singularity at mass point with infinite gravitational redshift

$$1+Z_{\mathrm{g}}=\mathrm{e}^{\vartheta(r)}|_{r\to0}\to\infty \tag{79}$$

but no coordinate singularity at Schwarzschild radius $r_{\mathrm{S}}=2GM/c^2$. So that, it can only allow the appearance of a “dot-like” black hole, rather than the Schwarzschild’s surrounded by the event horizon.

Know that, Schwarzschild black hole is alleged from Schwarzschild metric that has a radial quantity r never being reasonably identified, and a zero curvature $R_{nl}=0$ that is by construction a spacetime devoid of all matter [8]. However, in measured space calibrated by Hubble radius $\Re$, the non-zero curvature determines that the static metric field around mass M can only have one unique physical singularity. As $\Re \to \infty$ and $r >> r_S$, it gives

$$\mathrm{e}^{-2\vartheta(r)} \approx 1-\frac{2GM}{c^2 r}\left(1-\frac{GM}{c^2 r}\right) \approx 1-\frac{GM}{c^2 r\mathrm{e}^{\vartheta}} \tag{80}$$

So, by transformation of $r \to r' = r\mathrm{e}^{\vartheta}$, metric (78) reduces to Schwarzschild solution ($\mathrm{d}\xi^2$ is absorbed into $\mathrm{d}s^2$)

$$\mathrm{d}s^2 = -c^2\left(1-\frac{2GM}{c^2 r}\right)\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) \tag{81}$$

From this, it is clear to see that the pseudo-Minkowski manifold does not support Schwarzschild black hole, thus avoiding the space fragmentation caused by the event horizon and the logic confusion by this fragmentation.

6 Conclusion

To sum up, unlike appending geometric constraint beyond the perceptible dimensions, we attribute the remodeling of pseudo-Minkowski space to an endogenous requirement that the interaction has finite propagation velocity and force-range. Only then can it ensure that the remodeled relativistic world is a completely autonomous motion system of matter that follows the endogenous logic rules, but is not influenced or constrained by "extrinsic" factors. As a result, working in the world, the optimized transformation not only exhibits stronger self-consistency and inclusiveness, but also provides quantitative explanations for the cosmological issues. It is intriguing that so much is accommodated by adhering to a basic judgment of nature that everything existing should be detectable rather than physically isolatable, otherwise, this so-called "existence" would be meaningless.

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